HOMEWORK 9
MATH 430

Problem 1. *Prove the Chinese Remainder Theorem: If* $d_1, ..., d_n$ *are relatively prime natural numbers, and* $a_1, ..., a_n$ *are such that for all* i*,* $a_i < d_i$*, then there is some* c*, such that for all* i*,* $c = a_i \mathit{mod}\, d_i$*.*

Recall that in class we defined a formula $\phi^*_{prime}(n, p)$ in a Σ_1 form, such that $\mathfrak{A} \models \phi^*_{prime}[n, p]$ iff p is the n-th prime. Here $\mathfrak{A} = (\mathbb{N}, 0, S, +, \cdot, <)$ is the standard model of PA.

Problem 2. *Show that* $\phi^*_{prime}(n, p)$ *is* Δ_1 *by writing a* Π_1 *formula and showing that it is equivalent to* $\phi^*_{prime}(n, p)$*.*

Problem 3. *Show that any model* $\mathfrak{B}$ *of PA is an end-extension of the standard model* $\mathfrak{A}$*. I.e. show that there is a one-to-one function* $h : \mathbb{N} \to |\mathfrak{B}|$*, such that* h *is homomorphism (see the definition on page 94 in the book), and for every* $b <_{\mathbb{B}} c$*, if* $c \in \operatorname{ran}(h)$*, then* $b \in \operatorname{ran}(h)$*.*

Problem 4. *Suppose that* ϕ *is a* Δ_0*-formula, such that* $\mathfrak{A} \models \phi$*. Show that any model* $\mathfrak{B}$ *of PA is satisfies* ϕ*. Conclude that if* ϕ *is a* Δ_0*-formula, then* $\mathfrak{A} \models \phi$ *iff* $PA \models \phi$*.*